MATH 147 Review: Parametrization of Arcs/Lines

Facts to Know

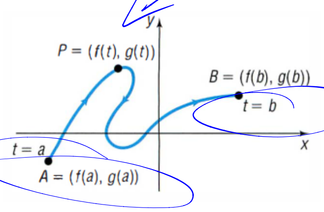
Plane curve

$\mathbb{R}^2$

"closed"

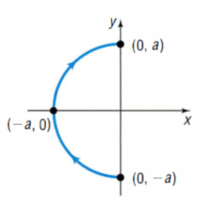
"simple"

Parametrization



$$\begin{cases} x = f(t) \\ y = g(t) \\ t \in [\alpha, \beta] \end{cases}$$

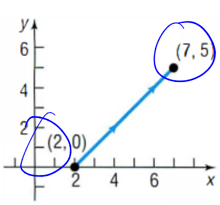
Circle arc



$$\begin{cases} x(t) = a\cos\left(-t + -\frac{\pi}{2}\right) \\ y(t) = a\sin\left(-t + -\frac{\pi}{2}\right) \end{cases}$$

$t \in [0, \pi]$

Line segment



end

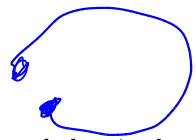
start

$$\begin{cases} x(t) = (1-t)2 + t7 \\ \quad = 2 - 2t + 7t = 5t + 2 \\ y(t) = (1-t)0 + t5 \end{cases}$$

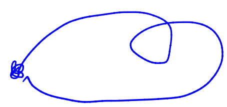
$t \in [0, 1]$

Examples

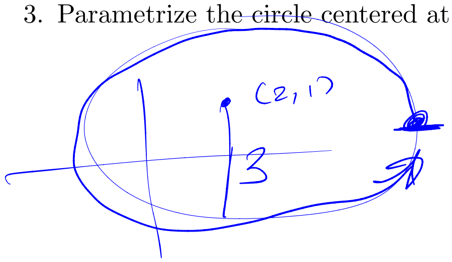
1. Draw a plane curve that is simple and that is not closed.



2. Draw a plane curve that is not simple and that is closed.



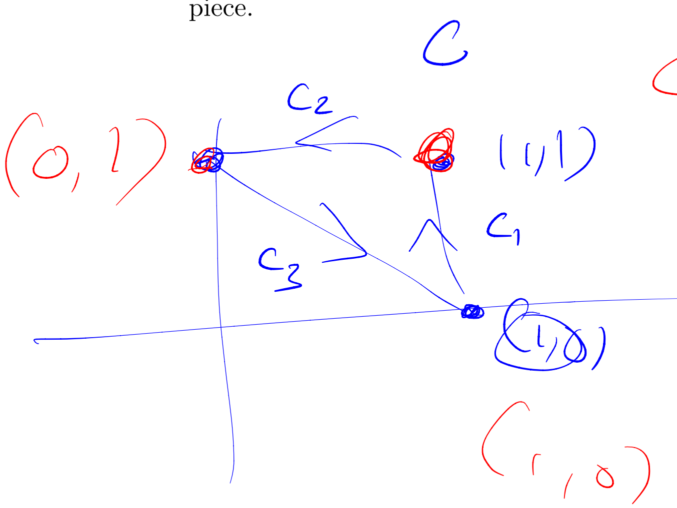
3. Parametrize the ~~circle~~ centered at $(2, 1)$ with radius 3 with a counter-clockwise orientation.



$$\begin{cases} x(t) = 3\cos(t) + 2 \\ y(t) = 3\sin(t) + 1 \end{cases}$$

$$t \in [0, 2\pi]$$

4. Parametrize the triangle with vertices located at $(1, 1)$, $(1, 0)$, and $(0, 1)$ with a counter-clockwise orientation. You are allowed to partition the curve into pieces and parametrize each piece.



$$C_1 \begin{cases} x_1(t) = (1-t)1 + t\cdot 1 \\ y_1(t) = (1-t)\cdot 0 + t\cdot 1 \end{cases}$$

$$C_2 \begin{cases} x_2(t) = (1-t)1 + t\cdot 0 \\ y_2(t) = (1-t)1 + t\cdot 1 \end{cases}$$

$$C_3 \begin{cases} x_3(t) = (1-t)\cdot 0 + t\cdot 1 \\ y_3(t) = (1-t)\cdot 1 + t\cdot 0 \end{cases}$$

$$t \in [0, 1]$$